2. **Set language and notation**	• use language, notation and Venn diagrams to describe sets and represent relationships between sets Definition of sets: e.g. $A = \{x : x$ is a natural number$\}$ $B = \{(x, y): y = mx + c\}$ $C = \{x : a \leqslant x \leqslant b\}$ $D = \{a, b, c...\}$	Includes using Venn diagrams to solve problems. Notation: Number of elements in set A $\quad$ n(A) "... is an element of ..." $\quad$ $\in$ "... is not an element of ..." $\quad$ $\notin$ Complement of set A $\quad$ A' The empty set $\quad$ $\varnothing$ Universal set $\quad$ $\mathscr{E}$ A is a subset of B $\quad$ $A \subseteq B$ A is a proper subset of B $\quad$ $A \subset B$ A is not a subset of B $\quad$ $A \nsubseteq B$ A is not a proper subset of B $\quad$ $A \not\subset B$ Union of A and B $\quad$ $A \cup B$ Intersection of A and B $\quad$ $A \cap B$

Definitions

1. A set is a collection of objects such as letters of the alphabet, people, etc. The objects in a set are called members or elements of that set.

2. A finite set is a set which contains a countable number of elements.

3. An infinite set is a set which contains an uncountable number of elements.

4. A universal set ξ is a set which contains all the available elements.

5. The empty set $\varnothing$ or null set { } is a set which contains no elements.
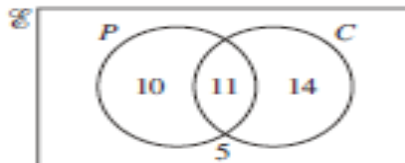
Specifications of Sets

6. A set may be specified by listing all its members.
    This is only for finite sets. We list names of elements of a set, separate them by commas and enclose them in brackets, e.g. {2, 3, 5, 7}.

7. A set may be specified by stating a property of its elements,
    e.g. {x: x is an even number greater than 3}.

For understanding

https://www.youtube.com/watch?v=2TMu2cr9oDk

https://www.youtube.com/watch?v=xwK--rNDI9E

8. A set may be specified by the use of a Venn diagram.
   e.g.

For example, the Venn diagram above represents
ξ = {students in the class},
P = {students who study Physics},
C = {students who study Chemistry}.

From the Venn diagram,
10 students study Physics only,
14 students study Chemistry only,
11 students study both Physics and Chemistry,
5 students do not study either Physics or Chemistry.

Elements of a Set

9. a $\in Q$ means that a is an element of Q.
   b $\notin Q$ means that b is not an element of Q.

10. n(A) denotes the number of elements in set A.

Equal Sets

11. If two sets contain the exact same elements, we say that the two sets are equal sets. For example, if $A = \{1, 2, 3\}$, $B = \{3, 1, 2\}$ and $C = \{a, b, c\}$, then A and B are equal sets but A and C are not equal sets.

Subsets

12. $A \subseteq B$ means that A is a subset of B.
    Every element of set A is also an element of set B.
13. $A \subset B$ means that A is a proper subset of B.
    Every element of set A is also an element of set B, but A cannot be equal to B.
14. $A \not\subseteq B$ means A is not a subset of B.
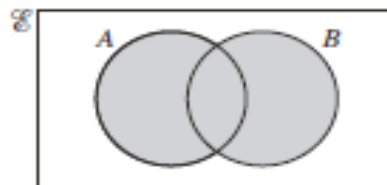15. $A \not\subset B$ means A is not a proper subset of B.

Complement Sets

16. A' denotes the complement of a set A relative to a universal set ξ.
    It is the set of all elements in ξ except those in A.
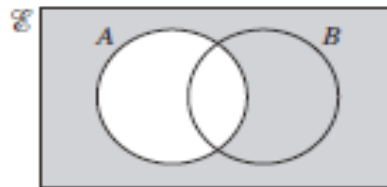
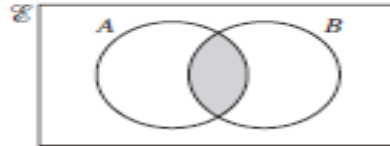The shaded region in the diagram shows A'.

Union of Two Sets

17. The union of two sets A and B, denoted as $A \cup B$, is the set of elements which belong to set A or set B or both.
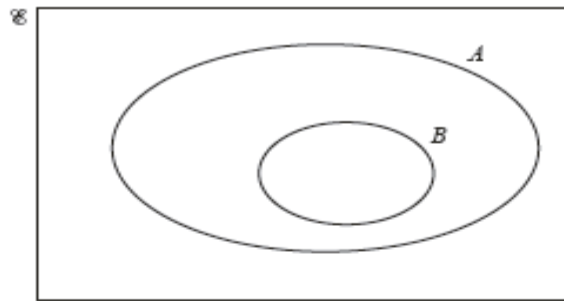

The shaded region in the diagram shows $A \cup B$.

Intersection of Two Sets

18. The intersection of two sets A and B, denoted as $A \cap B$, is the set of elements which belong to both set A and set B.

The shaded region in the diagram shows $A \cap B$.

M/J19/11/20

1 (a) On the Venn diagram, shade the region represented by $B' \cap A$.
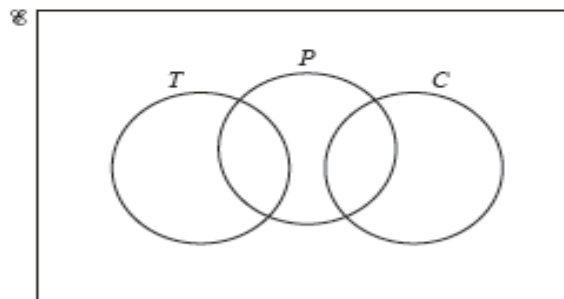
[1]

(b) Here is some information about 100 people who visit a café.

32 drink coffee (C)
40 drink tea (T)
50 eat a pastry (P)
18 drink coffee and eat a pastry
21 drink tea and eat a pastry

(i) Complete the Venn diagram to show this information for the 100 people.

[2]

(ii) Find $n(T \cup P \cup C)'$.

.... [1]

2 (a) Use set notation to describe the shaded region in the Venn diagram.

M/J19/12/10

.... [1]
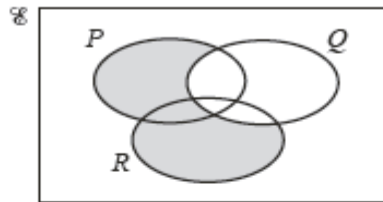
(b) $\mathscr{E} = \{x : x \text{ is a positive number}\}$
$A = \{x : 9 < x < 10\}$
$B = \{x : x \text{ is an irrational number}\}$

Write down an element of $A \cap B$.

.... [2]

3   The sets A, B and C are shown in the Venn diagram.   SP18/02/5(b and c)

$\mathscr{E} = \{\, x : x \text{ is an integer, } 1 \leqslant x \leqslant 18 \,\}$
$A = \{\, x : x \text{ is an even number} \,\}$
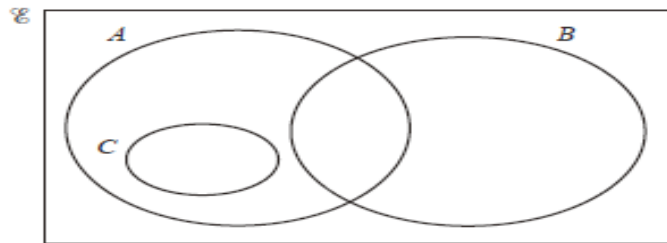$B = \{\, x : x \text{ is a multiple of 5} \,\}$

(i)   Find n($A \cup B$).

Answer .... [1]

(ii)   (a)   Given that $A \cap B' \cap C = \{2, 6, 14, 18\}$, list the members of C.

Answer .... [1]

(b)   Describe the set C in words.

Answer $C = \{\, x : x \text{ is }$ .... $\} \, [1]$

(c)   A school offers piano lessons and flute lessons to a group of 50 children.

Of these children, 28 attend piano lessons
               17 attend flute lessons
               12 attend neither piano lessons nor flute lessons.

By drawing a Venn diagram, or otherwise, find the number of children who attend only the piano lessons.

Answer ....
[2]

4     $\mathscr{E} = \{0, 1, 2, 3, 4, 5, 6\}$ O/N18/12/22
$P = \{x : x = 0, 1, 2\}$
$Q = \{y : y = 0, 2\}$

(a) List the members of $P \cap Q$.

Answer .... [1]

(b) Find $n(P' \cup Q)$.

Answer .... [1]

(c) $R = \{z : z = 2x + y, \, x \in P, \, y \in Q\}$

List the members of R.

Answer .... [2]

5 (a) $\mathscr{E} = \{x : x \text{ is an integer } 1 \leqslant x \leqslant 10\}$ O/N18/21/4
$A = \{x : x \text{ is a factor of } 20\}$
$B = \{x : x \text{ is a multiple of } 4\}$

(i) Complete the Venn diagram.
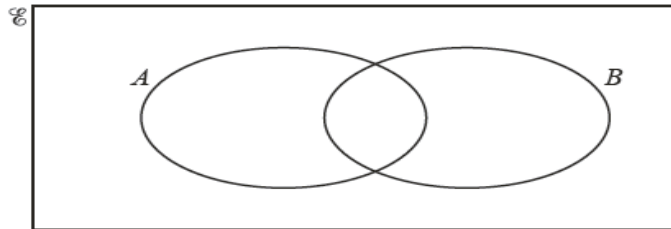
[2]

(ii) State $n(A \cup B)$.

Answer .... [1]

(iii) Describe in words the set $A \cap B'$.

Answer .... [1]

(b) 30 people are asked what type of fruit they like.
Of these people,

- 5 say they like both oranges and bananas
- 12 say they like oranges
- 8 say they like neither oranges nor bananas.

(i) By drawing a Venn diagram, or otherwise, find the number of people who like bananas but not oranges.

Answer .... [2]

(ii) Two of the 30 people are selected at random.

Find the probability that they both like oranges but not bananas.

Answer .... [2]

6 (a) Use set notation to describe the shaded region in the Venn diagram.
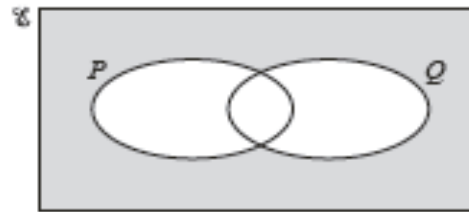
M/J18/21/1(a and b)

Answer .... [1]

(b) $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
$A = \{x : x \text{ is a factor of } 12\}$
$B = \{x : x \text{ is a multiple of } 2\}$
$C = \{x : x \text{ is a square number}\}$

(i) Show this information on the Venn diagram below.
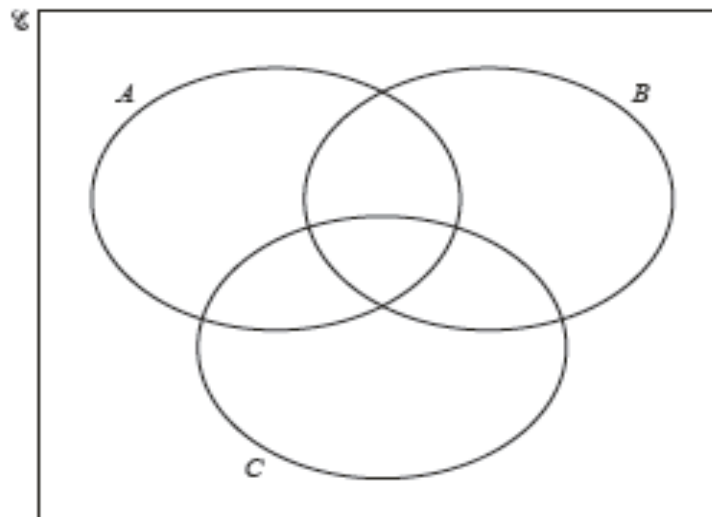
[2]

(ii) Find $n(A \cap B)$.

Answer .... [1]

(iii) Find $n(A \cap (B \cup C)')$.

Answer .... [1]

(iv) One subset in the Venn diagram in **part (b)(i)** has no elements.

Use set notation to describe this subset.

Answer .... [1]

7 **(a)** $\mathscr{E} = \{x : x \text{ is an integer } 1 \leqslant x \leqslant 18\}$ M/J18/22/4(a)
$A = \{x : x \text{ is a prime number}\}$
$B = \{1, 2, 3, 4, 6, 9, 12, 18\}$
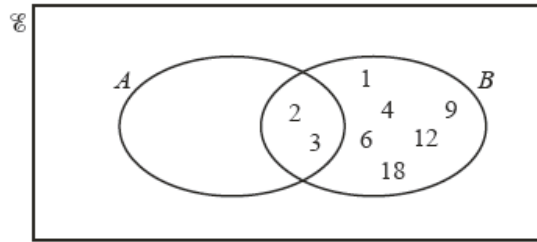
(i) Complete the Venn diagram to illustrate this information. [1]

(ii) Complete the description of the set B.

Answer $B = \{ x : x \text{ is a factor of } .... \}$ [1]
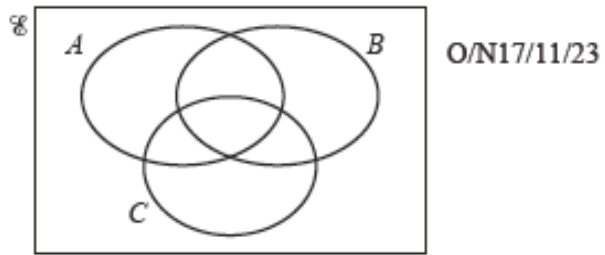
(iii) Find n$(A \cup B)$.

Answer .... [1]

(iv) List the elements of $A' \cap B$.

Answer .... [1]

8 (a) In the Venn diagram, shade the region which represents the subset $A' \cap B \cap C$.

O/N17/11/23

[1]

(b) $P = \{1, 4\}$
$Q = \{-1, 1, 2\}$
$R = \left\{ \dfrac{x}{y} : x \in P, y \in Q \right\}$

(i) Find $n(P \cup Q)$.

Answer .... [1]

(ii) List the members of R.

Answer .... [2]

9 (a) $\mathscr{E} = \{x : x$ is an integer and $10 \leqslant x \leqslant 20\}$

   $A = \{x : x$ is an odd number$\}$ O/N17/22/6

   $B = \{x : x$ is a multiple of 5$\}$

   (i) Find $n(A \cap B)$.

   Answer .... [1]

   (ii) Find $A' \cup B$.

   Answer .... [1]

   (iii) A number, r, is chosen at random from $\mathscr{E}$.

   Find the probability that $r \in A \cup B$.

   Answer .... [1]

(b) In a survey, 40 people were asked what they had read that day.

- A total of 10 people had read a book
- A total of 24 people had read a newspaper
- 14 people had read neither a book nor a newspaper

    (i) By drawing a Venn diagram, or otherwise, find the number of people who had read both a book and a newspaper.

Answer .... [2]

    (ii) Two of the 10 people who had read a book are selected at random.

        Work out the probability that they had both read a book and a newspaper.

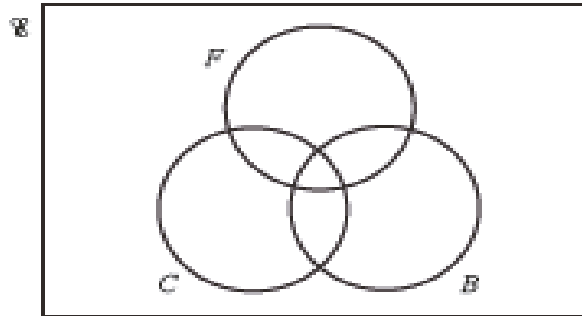*Answer* .... [2]

10 (a) In a sports club   24 members play basketball (B),
                                  28 play cricket (C),   M/J17/11/21

                                     16 play football (F),

                                       9 play basketball and cricket,

                                    11 play cricket and football and

                                    6 play basketball and football.

Five members play all three games and **eight** members play none of these games.

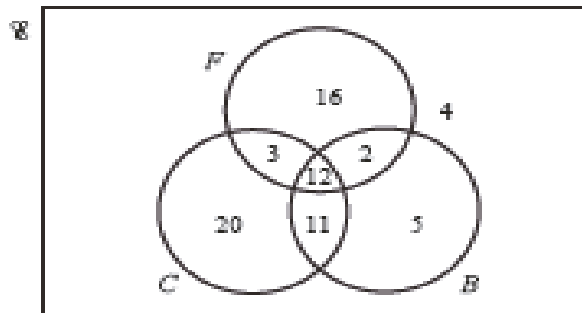**(i)** Complete the Venn diagram to show this information.

                                                                     [2]

(ii) Hence work out the **total** number of members in the club.

Answer .... [1]

(b) In another sports club, the number of members playing basketball (B), cricket (C) and football (F) are shown in the Venn diagram below.
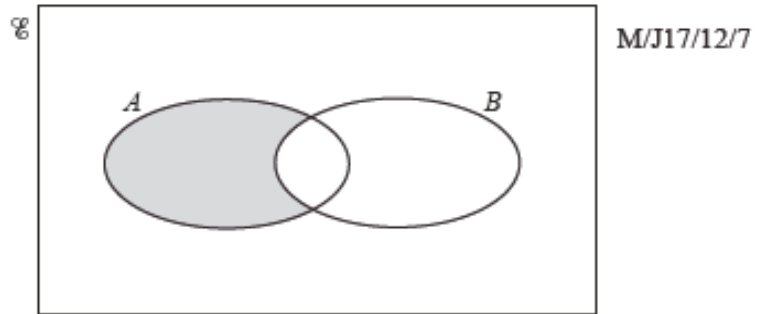

(i) Find n(F').

Answer .... [1]

(ii) Find n(($F \cup C$) ∩ B').

Answer .... [1]

11(a) Use set notation to describe the shaded set in the Venn diagram.

M/J17/12/7

Answer .... [1]

(b) Use set notation to complete the statement about sets C and D.
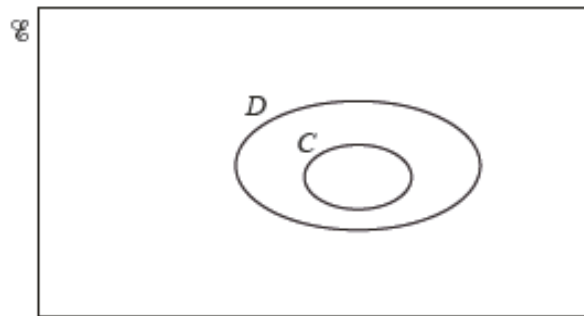
Answer C .... D [1]

12 (a) $\mathscr{E}$ = { 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96 }

   P = { x : x is an even number }   O/N16/11/12
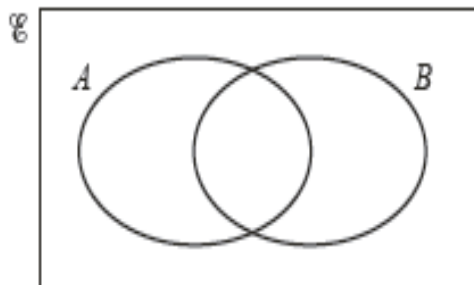
   Q = { x : x is a multiple of 3 }

   (i)   Find   n($P \cup Q$).

   Answer .... [1]

   (ii)   Given that $y \in \mathscr{E}$ and that y is a prime number, write down the value of y.

   Answer y = ... [1]

(b) In the Venn diagram, shade the region represented by $A' \cap B$.

[1]

13 **(a)** In the Venn diagram, shade the region which represents the subset $(A \cap B') \cup C$.

O/N16/12/13
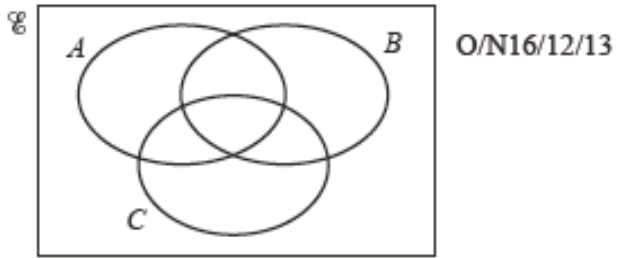
[1]

(b) In a group of 36 students,

23 study Spanish,
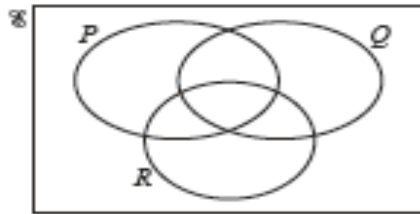17 study French,
4 study neither Spanish nor French.

By drawing a Venn diagram, or otherwise, find the number of students who study both Spanish and French.

Answer .... [2]

14 (a) In the Venn diagram, shade the region which represents the subset $(P \cup Q)' \cap R$.

M/J16/11/23 [1]

(b) $\mathscr{E} = \{ x : x$ is an integer and $22 \leqslant x \leqslant 33 \}$
$E = \{ x : x$ is an even number $\}$
$T = \{ x : x$ is a multiple of 3 $\}$
$F = \{ x : x$ is a multiple of 4 $\}$

(i) List the members of $T \cap F$.

Answer .... [1]

(ii) Find $n(E \cup T)$.

Answer .... [1]

(iii) Given that $y \in F' \cap E$, find one possible value of y.

Answer $y =$ .... [1]

15 **(a)** $\mathcal{E} = \{ 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 \}$
$A = \{ x : x \text{ is a prime number} \}$
$B = \{ x : x \text{ is an even number} \}$ MJ16/22/6(a)
$C = \{ x : x \text{ is a multiple of 5} \}$

   (i) List the members of the subsets

   (a) $B \cap C$,

   Answer .... [1]

   (b) $(A \cup B \cup C)'$,

   Answer .... [1]

   (c) $A \cap B'$.

   Answer .... [1]

   (ii) A number q is chosen at random from $\mathcal{E}$.

   Find the probability that $q \in A \cap B'$.
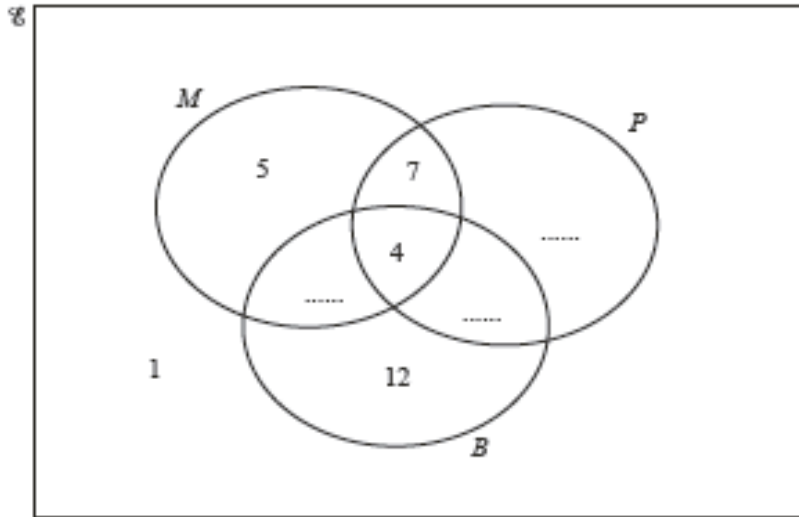
   Answer .... [1]

16 (a) Davinder asked some people if they ate mangoes, pineapples or bananas last week.

M = { people who ate mangoes }
P = { people who ate pineapples }
B = { people who ate bananas }

The Venn diagram shows some of the information.

19 people said they ate mangoes.
6 people said they ate **only** pineapples.
18 people said they ate **exactly two** of the three types of fruit.

(i) Write the three missing values in the Venn diagram. [3]

(ii) Find the total number of people Davinder asked.

.... [1]

(iii) Find $n(M \cap P)$.

.... [1]

(iv) One person is chosen at random from the people who ate mangoes.
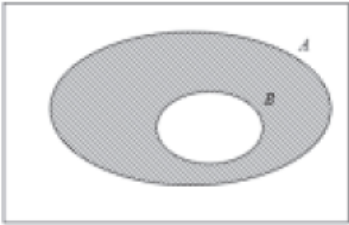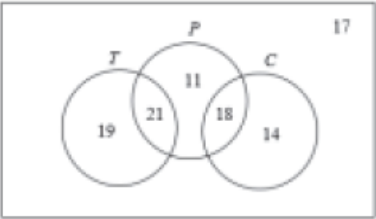
Write down the probability that this person also ate bananas.

.... [2]

Marking Scheme
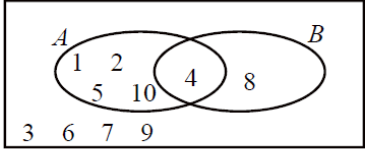
1(a)	Correct region shaded	1	
1(b)(i)	Correct Venn diagram.	2	M1 for 3, 4 or 5 subsets correct
1(b)(ii)	17	1	FT from *their* Venn diagram.
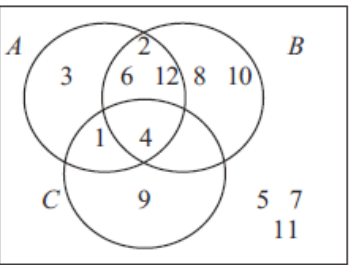
2(a)	$(P \cup R) \cap Q'$	1	
2(b)	Any irrational number in range $9 < x < 10$	2	B1 for any irrational number as answer or any number in range $9 < x < 10$ as answer

3(b)(i)	11		1	
3(b)(ii)(a)	4, 8, 12, 16		1	
3(b)(ii)(b)	x is a multiple of 4		1	
3(c)	21		2	M1 for n$(P \cup F)' = 12$
4(a)	0, 2	1		
4(b)	6	1		
4(c)	0, 2, 4, 6		2	B1 for 0, 2, 4, 6 plus extras (e.g. repeated 2 or 4; or an 8) or B1 for three of 0, 2, 4, 6 with no extras. or M1 for $2x + y$ clearly seen correctly evaluated for two or more valid values of x and y

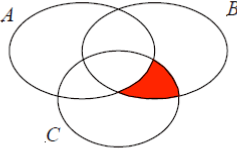| 5(a)(i)		2	**B1** for 8 or 9 numbers correctly placed or for 1, 2, 4, 5, 8, 10 correctly placed with no numbers placed incorrectly
5(a)(ii)	6	1	**FT** n($A \cup B$) from *their* Venn diagram
5(a)(iii)	Factors of 10 **oe**	1	
5(b)(i)	10	2	**B1** for Venn diagram with at least 3 numbers correct Or **M1** for 30 = 8 + 12 + x **oe**
5(b)(ii)	$\dfrac{42}{870}$ or $\dfrac{7}{145}$ **oe**	2	**M1** for $\dfrac{their\ 7}{30} \times \dfrac{their\ 6}{29}$ [×2] or **SC1** for answer $\dfrac{49}{900}$ **oe**, FT their Venn diagram

6(a)	$(P \cup Q)'$ or $P' \cap Q'$	1	
6b)(i)		2	**B1** for 8 or more correct
6(b)(ii)	4	1	FT *their* Venn diagram provided no repeated elements
6(b)(iii)	1	1	FT *their* Venn diagram provided no repeated elements
6(b)(iv)	$A' \cap B \cap C$	1	

7(a)(i)	Correctly completed Venn diagram	1	
7(a)(ii)	36	1	
7(a)(iii)	13	1	**FT** n($A \cup B$) from *their* Venn diagram provided no repeated elements in sets A and B
7(a)(iv)	1, 4, 6, 9, 12, 18	1	**FT** provided no repeated elements in sets A and B

8(a)		1	
8(b)(i)	4	1	
8b)(ii)	$\frac{1}{-1}, \frac{1}{1}, \frac{1}{2}, \frac{4}{-1}, \frac{4}{1}, \frac{4}{2}$ oe and isw	2	**C1** for 4 or 5 correct members

9(a)(i)	1	1	
9(a)(ii)	10, 12, 14, 15, 16, 18, 20	1	
9(a)(iii)	$\frac{7}{11}$ oe	1	
9(b)(i)	8	2	**M1** for $14 + 10 + 24 - x = 40$ oe or for correct Venn diagram with algebraic expressions. **Or B1** for Venn diagram with at least 3 numbers correct

9(b)(ii)	$\frac{28}{45}$ oe	2FT	**M1** for $\frac{their\,8}{k} \times \frac{their\,7}{k-1}[\times 2]$ where $k >$ their 8 Or **SC1** for $\left(\frac{their\,8}{10}\right)^2$
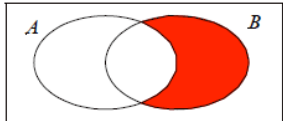
10(a)(i)	Correct Venn diagram 	2	B1 if 1 or 2 errors in the numbers
10(a)(ii)	55	1	
10(b)(i)	40	1	
10(b)(ii)	39	1	
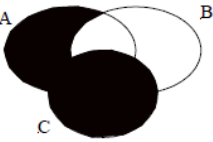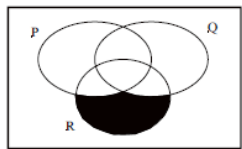
11(a)	$A \cap B'$ oe	1	
11(b)	$\subset$	1	

12 (a) (i)	9	1	
(ii)	89	1	
(b)		1	
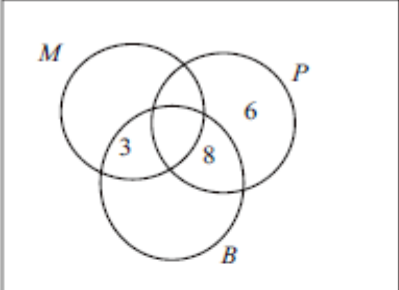
13 (a)		1	
(b)	8	2*	**M1** for $23 + 17 - (36 - 4)$ or **M1** for $23 - x + x + 17 - x + 4 = 36$ oe or **B1** for S ∩ F' = 15 or F ∩ S' = 9

14 (a)		1	
(b) (i)	24	1	
(ii)	8	1	
(iii)	22 or 26 or 30	1	

15 (a) (i)	(a) 10	1	
	(b) 9	1	
	(c) 3,5,7,11	1	
(ii)	$\frac{4}{11}$ oe isw	1ft	ft from *their* (a)(i)(c)

16	(a)	(i)		3	**B1** for each
		(ii)	46	1FT	**FT** 29 + *their* 3 values from **(a)**
		(iii)	11	1	
		(iv)	$\dfrac{7}{19}$ oe	2	**M1** for $\dfrac{n}{16 + their3}$ $(0 < n < (16 + their\ 3))$ or $\dfrac{4 + their\ 3}{k}$ $(k > (4 + their\ 3))$
	(b)	(i)	$\dfrac{9}{200}$ or 0.045	1	
		(ii)	10800	3	**M2** for ½ (900 + 1500) × 9 oe or **M1** for method of finding a relevant area
		(iii)	7.2	1FT	**FT** (*their* 10800) ÷ 1500

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